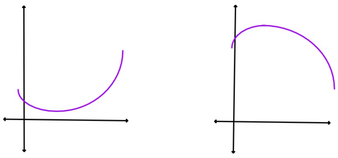
DO NOW

1. What is the difference between these two graphs?



2. Where do the tangent lines lie on each graph?

3. Which graph would you classify as concave upward?
   What about concave downward?

Page 1

4.4 Concavity and the Second Derivative Test

Concavity - curvature

Let f be differentiable on an open interval I.

1. If the graph of f is concave ***upward*** on I:
   - f lies above all tangent lines
   - ★ bowl opens up
2. If the graph of f is concave ***downward*** on I:
   - f lies below all tangent lines
   - ★ bowl open down

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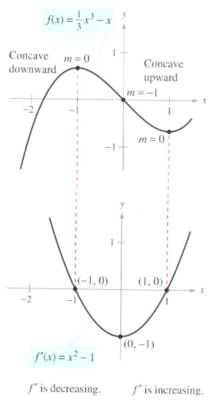
The concavity of f is related to the slope of its derivative.



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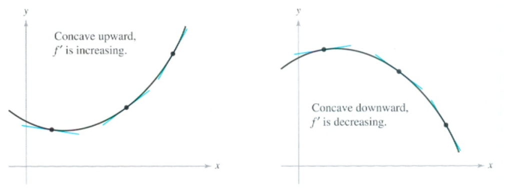
Definition of Concavity:

Let f be differentiable on an open interval I. The graph of f is:

Concave upward when f' is increasing.
Concave downward when f' is decreasing



(a) The graph of f lies above its tangent lines. **(b)** The graph of f lies below its tangent lines.

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Theorem: Test for Concavity

Let f be a function whose second derivative exists on an open interval I.

1. If $f''(x) > 0$ for all x in I, then concave upward
2. If $f''(x) < 0$ for all x in I, then concave downward.
3. If $f''(x) = 0$, there is no concavity

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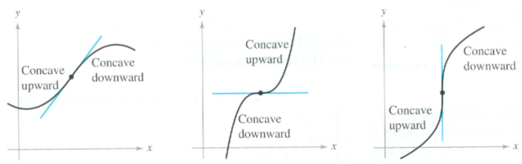
Point of Inflection:
- The function is continuous at c in the open interval.

-The graph has a tangent line at $(c, f(c))$

** where the concavity of f changes.
  —either from up to down
  or down to up



Note: The graph crosses its tangent line at a point of inflection.

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Theorem: Points of Inflection

If $(c, f(c))$ is a point of inflection of the graph of f, then either:

$f''(c) = 0$ or $f''(c)$ does not exist

Steps to investigate concavity:

1. Locate possible points of inflection. Set test intervals
2. Determine the SIGN of $f''(x)$ at one point in the test interval.
3. Determine concavity

Example: Find the points of inflection and discuss the concavity of the graph of the function: $f(x) = x^4 - 2x^3$

$f'(x) = 4x^3 - 6x^2$

$f''(x) = 12x^2 - 12x$

$12x(x-1) = 0$

$12x = 0$ or $x - 1 = 0$

$x = 0$ $x = 1$ ← possible points of inflection

$(-\infty, 0)$	$(0, 1)$	$(1, \infty)$
$f''(-1) = +$	$f''(\frac{1}{2}) = -$	$f''(2) = +$
concave up	concave down	concave up

$f(0) = 0$

$f(1) = 1 - 2 = -1$

points of inflection: $(0,0)$ and $(1,-1)$

HOMEWORK

pg 235; 1 - 6, 11, 12, 15, 17, 19, 24, 26, 27